Table 1 Sandia ARIES-II drops for UHLCADS

Drop no.	Test date	Time	q_0,[a] psf	V_0[b] kcas/ft/s		h_0,[c] msl	W,[d] lb	V_i,[e] ft/s	Results
1	5/12/83	12:17 p.m.	203	250	530	20,000	2341	30	Successful
2	6/9/83	10:32 a.m.	172	230	540	22,000	1600	25	Successful

[a] q_0 = dynamic pressure at release. [b] V_0 = velocity at release. [c] h_0 = altitude above sea level. [d] W = total vehicle weight. [e] V_i = impact velocity.

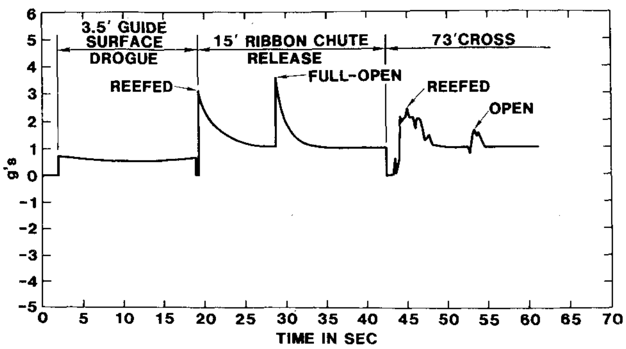

Fig. 2 Acceleration profile White Sands Test, June 9, 1983.

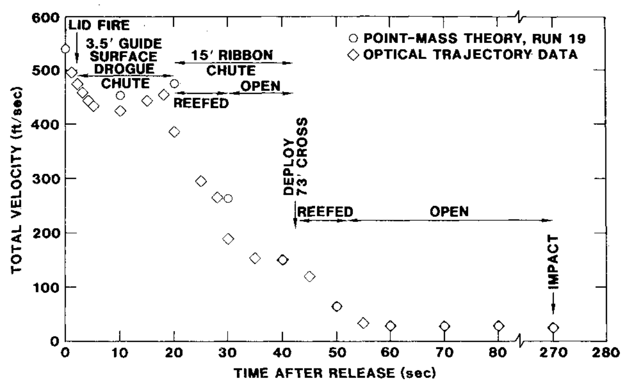

Fig. 3 ARIES-II drop No. 2.

method for the 3.5-ft guide surface drogue and 15-ft ribbon parachute. This eliminated the costly ball-lock load plate, and allowed staging by severing a double 5000-lb braided Kevlar pucker cord with a cable cutter fired by the clock timer in the vehicle. The deceleration record from the second drop is shown in Fig. 2. The positive load peaks shown in Fig. 2 are due to the aft parachute load force along the vehicle axis. The velocity decay with time after release, as obtained from phototheodolite tracking cameras, is shown in Fig. 3. Point-mass theoretical trajectory data shown as circles agree well with the diamond-shaped marks for the tracking data.

Conclusions

From two drop tests of the ARIES parachute system, it has been concluded that

1) The proposed 15-ft-diam ribbon first-stage and 73-ft-diam cross second-stage parachutes are well suited for the delivery application.

2) Impact velocity of the 2400-lb maximum weight container will be 30 ft/s at an altitude of 4700 ft mean sea level.

3) Maximum deceleration can be limited to slightly over 3 g's as requested.

Acknowledgments

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A Theorem on Swirl Loss in Propeller Wakes

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Introduction

THE wake of a propeller has tangential as well as longitudinal velocity relative to still air. The coarser the spiral the propeller carves in the air, the greater is the ratio of tangential to longitudinal wake velocity. As aircraft with propellers go faster, designers must use higher and higher ratios of pitch to diameter to keep the propeller tips below, or not too far above, the speed of sound. The higher thrusts required for higher speeds demand higher loading. All wake energy losses will rise, the swirl component more than the longitudinal component.

Swirl losses may appear particularly frightening because, as well as being a repository of kinetic energy, the swirling motion makes the wake a vortex, which, like any other vortex, has reduced pressure within itself. Added to the energy cost of creating the swirling motion, there appears to be a pressure drag due to the suction in the wake.[1] Fortunately, this is not true. In good approximation, the pressure drag is the way the swirl energy cost is paid for. The two do not add together.

The Proof

Commence with the general momentum theory of Glauert,[2] which he ascribes Joukowski. Glauert's Eq. (1.7) concerns a propeller with an infinite number of blades and gives the pressure depression at any radius r in the wake (Fig. 1). It is

$$p = (\rho/2)(V^2 - u^2) + \rho(\Omega - \omega/2)\omega r^2 \tag{1}$$

where p is the pressure in the wake minus the ambient pressure (p is always negative), ρ is the density of air, V the forward

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speed of the aircraft, u the speed of the slipstream relative to the aircraft, Ω the angular velocity of the propeller, and ω the angular velocity of the slipstream at radius r. Multiply the equation by u, rearrange it, and integrate it over the wake area to get

$$2\pi\rho\int_{r=0}^{r=R}\Omega\omega ur^3\mathrm{d}r-2\pi\int_{r=0}^{r=R}pur\mathrm{d}r$$

$$=\pi\rho\int_{r=0}^{r=R}(u^2-V^2+\omega^2r^2)ur\mathrm{d}r \tag{2}$$

where R is the radius of the edge of the wake. The first term on the left is shaft power, the second is the rate at which wake suction does work on the propeller—imagine that there is a piston racing backward, pulling on the far end of the wake—and the right side is the rate at which energy accumulates in the fluid. Note that this is energy in a coordinate system that moves with the propeller plane.

The radial pressure gradient in the wake is given by

$$\mathrm{d}p/\mathrm{d}r=\rho\omega^2r \tag{3}$$

so the integral

$$\pi\rho\int_{r=0}^{r=R}\omega^2r^2ur\mathrm{d}r$$

in Eq. (2), the rate of accumulation of swirl energy, can be replaced by

$$\pi\int_{r=0}^{r=R}(\mathrm{d}p/\mathrm{d}r)ur^2\mathrm{d}r$$

By means of the derivative of pur^2 with respect to r,

$$\mathrm{d}(pur^2)/\mathrm{d}r=(\mathrm{d}p/\mathrm{d}r)ur^2+p(\mathrm{d}u/\mathrm{d}r)r^2+2pur$$

the integral can finally be transformed by parts thus

$$\pi\rho\int_{r=0}^{r=R}\omega^2r^2ur\mathrm{d}r=\pi\int_{r=0}^{r=R}(\mathrm{d}p/\mathrm{d}r)ur^2\mathrm{d}r$$

$$=\pi pur^2\Big|_{r=0}^{r=R}-2\pi\int_{r=0}^{r=R}pur\mathrm{d}r-\pi\int_{r=0}^{r=R}p(\mathrm{d}u/\mathrm{d}r)r^2\mathrm{d}r \tag{4}$$

The term

$$2\pi pur^2\Big|_{r=0}^{r=R}$$

is zero for any real slipstream, because $r=0$ on the axis and $p=0$ at $r=R$. If u is uniform across the slipstream, $\mathrm{d}u/\mathrm{d}r=0$, except at $r=R$, where $p=0$, so the swirl energy comes entirely from work performed by wake suction. If the propeller is lightly loaded, this condition is approximately satisfied no matter what the distribution of thrust over the area of the propeller, because most of u is the forward speed of the aircraft. Nowhere does u differ much from V.

Putting the transformed integral into Eq. (2) gives

$$2\pi\rho\int_{r=0}^{r=R}\Omega\omega ur^3\mathrm{d}r=\pi\rho\int_{r=0}^{r=R}(u^2-V^2)ur\mathrm{d}r$$

$$-\pi\int_{r=0}^{r=R}p(\mathrm{d}u/\mathrm{d}r)r^2\mathrm{d}r \tag{5}$$

The shaft power is exactly equal to the rate of gain of longitudinal kinetic energy if u is constant across the wake. It is uninfluenced by swirl.

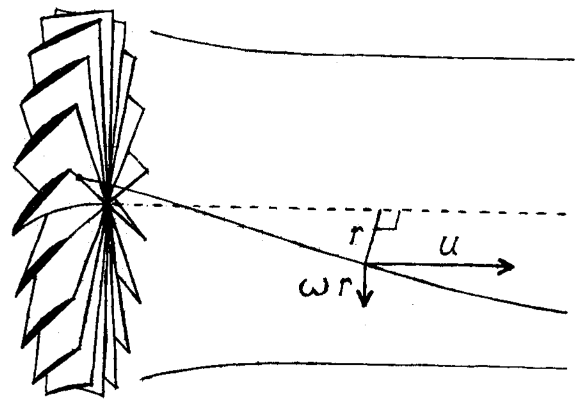

Fig. 1 A propeller and its swirling wake. In the theory it has infinitely many blades.

Of course, work done by the wake must come, in the end, from the propeller. The cost appears as reduced thrust T, given by Glauert's Eq. (1.10)

$$T=2\pi\rho\int_{r=0}^{r=R}(u-V)ur\mathrm{d}r+2\pi\int_{r=0}^{r=R}pr\mathrm{d}r \tag{6}$$

which is not limited to the case when u is constant across the wake. The second term is the reduction in thrust due to wake suction.

The propulsive efficiency is $TV/$(shaft power). For any particular distribution of u as a function of r, the only effect of swirl on efficiency is the reduction in thrust due to wake suction, and the small influence of the $(\mathrm{d}u)/(\mathrm{d}r)$ term in Eq. (4) on shaft power, an influence that vanishes if u is independent of r.

One should be disturbed by the piston that races backward to apply suction to the far end of the wake. But for most wakes the longitudinal kinetic energy of the fluid makes the piston unnecessary. If wake fluid is decelerated by climbing a pressure gradient to ambient pressure, its final longitudinal velocity will be positive, that is, downstream relative to the propeller, so long as $\rho u^2/2>-p$. Combining this with Eq. (1) gives

$$\rho\Omega wr^2+\rho V^2/2>\rho\omega^2r^2/2 \tag{7}$$

So long as the energy in a unit volume of fluid, in coordinates moving with the propeller plane, as it approaches the propeller (or windmill), plus the energy given it by the propeller, exceeds the swirl energy it has in the wake, the wake can be maintained without making the conceptual piston into a real low-pressure sink.

Coaxial Pairs of Propellers

Since nothing in the theorem requires that the angular velocity of the propeller be independent of radius, it would apply to a nested set of annular fans in the propeller plane, each spinning at a different angular velocity, and also to a contraprop, the pitch and chord of whose two members were imperfectly balanced against each other to eliminate swirl. By a simple generalization of the derivation of Eq. (1), given in the Appendix, it can be shown that Ω_{cp}, the effective angular velocity function of a coaxial pair, is given by

$$\Omega_{cp}=(\Omega_I-\Omega_{II})(\omega_br_b^2)/(\omega r^2)+\Omega_{II} \tag{8}$$

where Ω_I and Ω_{II} are the angular velocities of the first and second propellers, r_b is radius in the planes of the propellers, which are assumed to be close together, and ω_b is the swirl angular velocity between the propellers. Ω_{cp} is the radial distribution of the angular velocities of nested fans that would produce the same wake as the coaxial pair of propellers. For a swirlless counterrotating pair, $\omega=0$, $\Omega_{cp}=\infty$. If Ω_I or $\Omega_{II}=0$, the front or rear propeller becomes a set of fixed vanes that can pre- or deswirl the flow.

Unlike Eq. (1), Eq. (8) is not entirely in terms of wake quantities. The term $\omega_b r_b^2$, which is the angular momentum of a unit volume of fluid between the propellers at radius r_b, on the same streamline as r in the wake, cannot be inferred from the flow in the wake. The second propeller makes its own arbitrary contribution to the swirl, which renders wake swirl useless as evidence of the swirl between the propellers.

Conclusions

If the longitudinal component of velocity is uniform across a propeller wake, the swirl energy in the wake is exactly paid for by the work done by wake suction. For lightly loaded propellers the longitudinal velocity is always fairly uniform, and the relation between swirl energy and wake suction is a good approximation. Either swirl energy or wake suction may be used to measure swirl loss, but not both at once. It is not their sum.

Appendix

Let the propellers be close enough together so one can ignore the convergence of the flow between them and take the longitudinal component of velocity on any streamline as constant from one propeller to the next. The swirl component and the pressure can change at each propeller. Swirl angular velocity between the propellers is ω_b, that immediately after the second propeller is ω_a (the subscripts standing for between and after). The pressure jump across the upstream propeller is p'_I, across the downstream propeller p'_{II}, across both propellers $p'=p'_I+p'_{II}$.

Applying Bernoulli's equation to the velocity of the fluid relative to a point fixed to the upstream propeller at radius r_b, and rotating with it, relates the pressure increase across the propeller disk to the angular velocity of the propeller and the swirl downstream of it, thus

$$p'_I=(\rho/2)[\Omega_I^2-(\Omega_I-\omega_b)^2]r_b^2$$

$$=\rho(\Omega_I-\omega_b/2)\omega_b r_b^2 \qquad \text{(A1)}$$

Because the longitudinal velocity is the same up and downstream of the propeller, it does not appear in the equation.

The swirl component of the velocity of the fluid entering the second propeller relative to that of the propeller is $(\omega_b-\Omega_{II})r_b$, and of that leaving it is $(\omega_a-\Omega_{II})r_b$. By the same Bernoulli argument, the pressure rise across the downstream propeller is

$$p'_{II}=(\rho/2)[(\Omega_{II}-\omega_b)^2-(\Omega_{II}-\omega_a)^2]r_b^2 \qquad \text{(A2)}$$

so

$$p'=p'_I+p'_{II}=\rho[(\Omega_I-\Omega_{II})w_b+\Omega_{II}w_a-\omega_a^2/2]r_b^2 \qquad \text{(A3)}$$

Applying Bernoulli's theorem separately to the flow that accelerates from infinity to the upstream face of a propeller, and to the flow that accelerates from the downstream face to infinity gives

$$p=(\rho/2)(V^2-u^2+\omega_a^2 r_b^2-\omega^2 r^2)+p' \qquad \text{(A4)}$$

Combining this equation with Eq. (A3) gives

$$p=(\rho/2)(V^2-u^2)+\rho\{[(\Omega_I-\Omega_{II})\omega_b+\Omega_{II}\omega_a]r_b^2-\omega r^2/2\} \qquad \text{(A5)}$$

which reduces to Eq. (1) if either propeller is missing, that is, if $\omega_b=0$ or $\omega_b=\omega_a$.

Using the conservation of angular momentum relation

$$\omega_a r_b^2=\omega r^2 \qquad \text{(A6)}$$

the equation can be transformed into

$$p=(\rho/2)(V^2-u^2)+\rho[(\Omega_I-\Omega_{II})\omega_b r_b^2+(\Omega_{II}-\omega/2)\omega r^2] \qquad \text{(A7)}$$

which can be reduced to Eq. (1) by the substitution

$$\Omega_{cp}=(\Omega_I-\Omega_{II})(\omega_b r_b^2)/(\omega r^2)+\Omega_{II} \qquad \text{(A8)}$$

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The Unsuitability of Ellipsoids as Test Cases for Line-Source Methods

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RECENTLY interest has reawakened in calculating axisymmetric potential flow about bodies of revolution by means of line-source distributions on the symmetry axis.[1,2] This technique was first put forward by von Kármán.[3] The general procedure is to divide a portion of the x-axis inside the body into a number of line-source elements, over each of which the source density has a known variation: constant, linear, quadratic, etc. The combined stream function of the axial uniform stream plus the line-source distribution is set equal to zero at a number of points on the body profile, which yields a set of linear equations for the parameters defining the axial source distribution.

There are two problem areas for such methods. First, as von Kármán warned,[3] not every body can be represented by an axial source distribution. Second, the line source cannot extend to the ends of the body at finite strength or the velocities there will be infinite. Thus, the ends of the source distribution must be inset some distance from the ends of the body, and determination of this inset distance is both important and nonstraightforward. To address these problems and to demonstrate the accuracy of their methods, investigators com-

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